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MAR 75 J E YATES, G SANDRI F44620-75-C-0010

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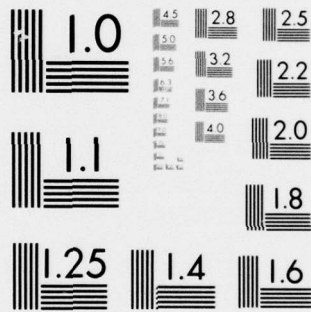
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER <b>AFOSR TR-77-0687</b>	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) <b>BERNOULLI ENTHALPY: A FUNDAMENTAL CONCEPT IN THE THEORY OF SOUND,</b>	5. TYPE OF REPORT & PERIOD COVERED <b>INTERIM</b> <b>ARAP-236</b>	
7. AUTHOR(s) <b>JOHN E. YATES Guido/Sandri</b>	6. PERFORMING ORG. REPORT NUMBER <b>ARAP Report No 236</b>	
9. PERFORMING ORGANIZATION NAME AND ADDRESS <b>AERONAUTICAL RESEARCH ASSOCIATES OF PRINCETON, INC 50 WASHINGTON ROAD, PRINCETON, N. J. 08540</b>	8. CONTRACT OR GRANT NUMBER(s) <b>F44620-75-C-0010</b>	
11. CONTROLLING OFFICE NAME AND ADDRESS <b>AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA 1400 WILSON BOULEVARD ARLINGTON, VIRGINIA 22209</b>	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS <b>9782-04 61102F</b>	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	12. REPORT DATE <b>March 1975</b>	
	13. NUMBER OF PAGES <b>12</b> <b>14p.</b>	
	15. SECURITY CLASS. (of this report) <b>UNCLASSIFIED</b>	
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE		
16. DISTRIBUTION STATEMENT (of this Report)  <b>Approved for public release; distribution unlimited.</b>		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES <b>PROCEEDING AIAA Aero-Acoustics Conference 2nd Hampton, VA 24-26 March 1975 AIAA Paper 75-439</b>		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) <b>SOUND THEORY BERNOULLI ENTHALPY SCATTERING PSEUDOSOUND</b>		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <b>A general theory of aerodynamic sound is developed. The basic equations of fluid mechanics are expressed in terms of velocity, enthalpy and entropy, and the velocity field is separated into irrotational and rotational parts with the latter being incompressible. By analogy with the description of sound in a homentropic-irrotational flow, the radiative part of the sound field is characterized by the velocity potential, <math>\phi</math>. A fundamental new concept, Bernoulli enthalpy, <math>\gamma</math>, is introduced, again by analogy with homentropic-irrotational flow. It is shown that the "source" of the radiative sound field is the substantive</b>		

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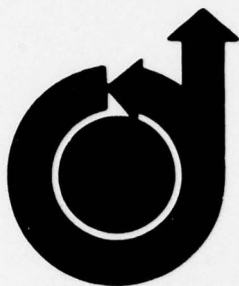
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# **AIAA 2nd Aero-Acoustics Conference**

HAMPTON, VA. / MARCH 24-26, 1975

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BERNOULLI ENTHALPY  
A FUNDAMENTAL CONCEPT IN THE THEORY OF SOUND

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Abstract

A general theory of aerodynamic sound is developed. The basic equations of fluid mechanics are expressed in terms of velocity, enthalpy and entropy, and the velocity field is separated into irrotational and rotational parts with the latter being incompressible. By analogy with the description of sound in a homentropic-irrotational flow, the radiative part of the sound field is characterized by the velocity potential,  $\phi$ . A fundamental new concept, Bernoulli enthalpy,  $H$ , is introduced, again by analogy with homentropic-irrotational flow. It is shown that the "source" of the radiative sound field is the substantive rate of change of the Bernoulli enthalpy. The detailed kinematic and thermodynamic mechanisms of sound production and interaction are ultimately contained in the source of the Bernoulli  $H$  field. The latter satisfies a Poisson equation subject to the boundary condition that the  $H$  field acquires appropriate constant value(s) when the vorticity and the entropy gradient vanish. Two basic conceptual issues of the modern theory of sound are resolved. First, the radiative sound is clearly separated from the source of sound with each part characterized by its own scalar field. Second, the Bernoulli source of sound is compact; i.e., it is confined to the region of rotational non-homentropic flow. The near field sound pressure is in general a nonlinear function of the  $\phi$  and  $H$  fields and is not separable except in the linear case.

A detailed discussion and interpretation of the general theory is given with emphasis on the production of sound by turbulent flows. It is found that an important dipole source of the Bernoulli field can result by passing turbulence through a highly curved potential flow. The dominant source of Bernoulli enthalpy in low-speed noncurved flows is the classic Lighthill quadrupole (1,2,3).

An analogy between the present theory and the pseudosound concept of Ribner (4) is given. The  $H$  field is analogous to Ribner's pseudosound, while his acoustic field is defined in terms of our velocity potential,  $\phi$ . The experimentalist is challenged to measure the two parts of the pressure by measuring the separate rotational and irrotational components of the velocity field.

The linear problem of sound interacting with a mean shear flow is formulated with the new theory. It is compared

briefly with the formulation of the same problem based on the Landahl-Lilley equation (5). It is shown that the interpretation of the basic equations in terms of acoustic and vortical modes (in the homentropic case) can lead to a linear coupling between the sound and vortical flow. This mechanism is eliminated in the third-order Landahl-Lilley formulation where the acoustic and vortical modes are coupled into a single pressure variable that is termed acoustic.

A numerical example of the interactive Bernoulli field is given for the case of a plane wave impinging on a Gaussian plane jet. At small incidence angles to the jet axis, the Bernoulli source has the appearance of a dipole while at large angles it has the appearance of a monopole.

I. Introduction

Since the pioneering theoretical foundation of Lighthill (1,2,3), there have been many contributions to the modern theory of aerodynamic sound. Much of the work immediately following the classic Lighthill papers was aimed at refinements and/or extensions of his basic theory (e.g., 6,7) while others, including Lighthill himself, attempted direct application of the theory to the problem of estimating noise (8). It is not our intent here to give an extensive review of historical developments. Rather we focus on a few of the fundamental efforts that have attempted to clarify the basic conceptual issues in the theory of sound. In particular, we confine our remarks to the work of Lighthill (1,2,3,8), Ribner (4), Crow (9), Lilley (5) and Doak (10).

A basic conceptual problem with all of the modern sound theories is that no clear cut distinction has been made between radiative sound and sound source in the near field. Considerable discussion has evolved around the Lighthill "source." It is well known that all convective and refractive effects are disguised in the "source" while the radiative field is governed by the linear wave operator for a stationary medium. For this reason the basic Lighthill theory has been termed the "acoustic analogy." The equation is "exact" but the approximations and interpretations of it are not.

Ribner (4) made an attempt to separate "radiative" from "source" sound when he introduced the concept of pseudosound. By separating the pressure into two parts and postulating a Poisson equation for the near

field (pseudosound), he was able to derive a wave equation for the radiative part with the near field as source. This is closer to the "truth," as we see it, than any theory on the market. However, it is not appealing from the point of view that we would like to derive our field equations from fundamentals. One cannot be quite sure that the postulated pseudosound is the pressure that would actually be measured in the near field. Also, there is some question about the compactness of the pseudosound field (see below).

More recently, Lilley (5), following the approach of Phillips (11) and Landahl (12), has developed a single third-order equation for the pressure fluctuation. It presumably contains all convective and refractive effects on the left-hand side and all quadratic "source" terms due to turbulence and mean shear on the right-hand side. Our objection is that the equation mixes the acoustic mode with the vorticity mode (13). It does not really isolate the source of the radiative part of the acoustic field. The conclusion based on the Landahl-Lilley equation (5,10) that there can be no linear coupling between the acoustic pressure field and the turbulence is a direct result of the choice of dependent variable (in this case, pressure). The very mechanism whereby linear coupling can occur is buried in the left-hand side of the equation. In the present work we show that the acoustic mode can interact linearly with the mean shear flow and perturbation vorticity to produce radiative sound.

The more recent momentum potential approach of Doak (10) seems to further confuse the issue of "source" identification. Using the Helmholtz decomposition of the momentum flux, Doak derives a fourth-order equation for the momentum flux potential with the solenoidal part appearing on the right-hand side. He submits that the rotational-solenoidal part can be identified as the rotational part of the turbulent field and, hence, part of the "source." The most obvious objection to this approach is that the momentum flux can be rotational even though the velocity field is irrotational. Thus, via Doak's method, the description of sound in a compressible but otherwise homentropic-irrotational flow would require both a scalar potential and vector field. Furthermore, the theory would indicate that the vector field acts as a "source" for the momentum potential. This state of affairs is totally unacceptable in a region where there can be no aerodynamic sources of sound.

Another conceptual problem with modern sound theories is that the various "sources" that have been proposed are not compact; i.e., confined to the region of rotational nonhomentropic flow. This has not been a serious practical problem since one can usually resort to asymptotic analysis to localize the "source" in the region of turbulent flow. However, the issue has been discussed in several of the fundamental papers

with a great deal of mutual criticism. Lighthill (8, Appendix B) attacked Ribner for using a noncompact source, while Crow (9) criticized both theories for having noncompact sources apparently giving Ribner the edge for being the least compact. He uses the machinery of singular perturbation theory in an effort to derive the conditions under which one can use the Lighthill source. All of the early discussions of this issue point to the need of a theory with a true compact source of radiative sound.

Crow (9) made a modest attempt to construct such a theory. He used the familiar Helmholtz decomposition of the velocity field in terms of irrotational and solenoidal parts. He also wrote down an enthalpy expression,  $H$ , that is close to the Bernoulli enthalpy introduced in the present work. He was not successful, however, in isolating the acoustic from the source field except for the linear case. Also, he did not seem to recognize the essential compact nature of the  $H$  field.

In the present work we develop a general theory of sound in which the radiative and source fields are clearly separated and in which the source is compact. The point of departure is analogous to that of Crow (9). The two basic differences are in the choice of boundary conditions for the Helmholtz decomposition of the velocity field and the definition of the compact Bernoulli enthalpy field. An essential theme of the present work is that two scalar fields are necessary to describe the processes of radiation and production of sound without ambiguity.

## II. The General Theory of Sound

### Basic Equations

Consider the equations of motion for a viscous heat conducting gas. We choose the enthalpy  $h$  and entropy  $s$  to be the primary thermodynamic variables. The continuity, momentum and energy equations are:

$$\frac{Dh}{Dt} + a^2 \operatorname{div} \vec{v} = \frac{\gamma}{\rho} (\Phi - \dot{Q}) \quad (2.1)$$

$$\frac{D\vec{v}}{Dt} + \operatorname{grad} h = T \operatorname{grad} s + \frac{\operatorname{div} \vec{\tau}}{\rho} \quad (2.2)$$

$$\frac{Ds}{Dt} = \frac{1}{\rho T} (\Phi - \dot{Q}) \quad (2.3)$$

where

$$\Phi = \vec{\tau} : \operatorname{grad} \vec{v} \quad \text{viscous dissipation}$$

$$\dot{Q} = \operatorname{div} \vec{q} \quad \text{bulk heat addition}$$

$$\vec{q} = \text{heat flux vector}$$

$$\vec{\tau} = \text{viscous stress tensor}$$

(2.4)

Also,  $\rho$  and  $T$  are the density and temperature, respectively, with

$$\gamma = 1 + \frac{1}{\rho T} \frac{\partial p}{\partial s} \Big|_{\rho}$$

and

$$a^2 = \frac{\partial p}{\partial \rho} \Big|_s \quad (2.5)$$

The pressure is a function of  $h$  and  $s$ , through the equation of state

$$p = p(h, s) \quad (2.6)$$

We consider the flow situation shown in Fig. 1. Imagine the flow to be separated into regions  $H_1$ ,  $H_2$  and  $R$  by surfaces  $S_1$  and  $S_2$ . If the region  $R$  is roughly axisymmetric,  $H_1$  and  $H_2$  constitute a single region  $H$ . In regions  $H_1$  and  $H_2$  the flow is assumed to be inviscid, homentropic and irrotational. The region  $R$  is a turbulent shear flow in which there can be strong entropy fluctuations and viscosity and heat conduction need not be negligible effects. First, we review the situation in the region  $H_1$  (or  $H_2$ ) and then proceed to the analysis of region  $R$ .

#### Homentropic-Irrotational Flow

The region  $H_1$  (or  $H_2$ ) is quite familiar to the unsteady aerodynamicist and is described by the following equations:

$$\frac{Dh}{Dt} + a^2 \operatorname{div} \vec{v} = 0 \quad (2.7)$$

$$\frac{\partial \vec{v}}{\partial t} + \operatorname{grad} \left( \frac{v^2}{2} + h \right) = 0 \quad (2.8)$$

$$\operatorname{curl} \vec{v} = 0$$

$$s = s_{\infty} \quad (2.9)$$

$$p = p(h) \quad (2.10)$$

Also, the local sound speed is a function of  $h$  only. Since the flow is irrotational, there exists a velocity potential  $\phi$  such that

$$\vec{v} = \operatorname{grad} \phi \quad (2.11)$$

The continuity and momentum equations become

$$\frac{Dh}{Dt} + a^2 \nabla^2 \phi = 0 \quad (2.12)$$

and

$$\operatorname{grad} \left( h + \frac{\partial \phi}{\partial t} + \frac{|\operatorname{grad} \phi|^2}{2} \right) = 0 \quad (2.13)$$

The object in parentheses in (2.13) is a spatial constant within  $H_1$  (or  $H_2$ ). We can further argue from boundary conditions at  $\infty$  that it is also independent of time. To the unsteady aerodynamicist this object is known as the Bernoulli constant; i.e.,

$$h + \frac{\partial \phi}{\partial t} + \frac{1}{2} |\operatorname{grad} \phi|^2 = h_{\infty} + \frac{v_{\infty}^2}{2} = h_{\infty} \quad (2.14)$$

With this important result, we can immediately write down the nonlinear convective wave equation for  $\phi$ ; i.e.,

$$\frac{D}{Dt} \left( \frac{\partial \phi}{\partial t} + \frac{|\operatorname{grad} \phi|^2}{2} \right) - a^2 \nabla^2 \phi = 0 \quad (2.15)$$

and the pressure is given by the state equation

$$p = p \left( h_{\infty} + \frac{v_{\infty}^2}{2} - \frac{\partial \phi}{\partial t} - \frac{1}{2} |\operatorname{grad} \phi|^2 \right) \quad (2.16)$$

In region  $H_1$  (or  $H_2$ ), the propagation of sound is completely characterized by the single quasi-linear wave equation for  $\phi$ , (2.15). There are no aerodynamic sources of sound in the homentropic irrotational region. Sound can only be produced by the motion of an embedded unsteady surface in the flow. All of these results are well known and form the cornerstone of all unsteady potential aerodynamic theory.

Several other points can be made for the homentropic region. First, the  $\phi$  field in region  $H$  is completely determined from a knowledge of the normal velocity field on the bounding surface  $S$  (see Fig. 1), a fact pointed out long ago by Liepmann (14). From a practical standpoint, this surface can be very close to the edge of a turbulent jet, wake, boundary layer or what have you. Measured quantities on a surface in the "immediate" near field can be used to predict the far field.

A second point concerning the homentropic region is that pressure (or density) is not, in general, a convenient variable to describe the acoustic field. The inversion of the state relation (2.16) and solution for  $\phi$  in terms of  $p$  is not a simple task unless the fields are linear.\* Most of the sound theories presently in vogue have the notion of a linear sound field built into them and so can be cast in terms of pressure or density.

Finally, we point out that if the overall flow is curved due to the presence of solid bodies, then the curvature is included in the  $\phi$  field. The mean steady potential flow as well as the unsteady near and far fields are described by  $\phi$  alone in homentropic-irrotational flow.

#### Rotational-Turbulent Shear Flow

The region  $R$  presents a much more complicated flow picture than that in regions  $H_1$  (or  $H_2$ ). However, if we are attempting to describe acoustics, it seems most desirable to cling to the simple notion of a velocity potential. This we can do by using

\* We note that the resultant equation for  $\phi$  is of the form of the Eikonal equation of geometric wave theory or of the Hamilton-Jacobi equation (15).



the well-known Helmholtz decomposition of an arbitrary vector field into irrotational and rotational components (16). We let

$$\vec{v} = \text{grad } \phi + \vec{u} \quad (2.17)$$

where  $\phi$  has the same meaning as it did for homentropic-irrotational flow. The  $\vec{u}$  field is incompressible; i.e.,

$$\text{div } \vec{u} = 0 \quad (2.18)$$

Furthermore, we demand as a boundary condition that the field  $\vec{u}$  be zero if and only if the vorticity vanishes; i.e., if

$$\vec{\omega} = \text{curl } \vec{v} = \text{curl } \vec{u} \quad (2.19)$$

then

$$\vec{\omega} = 0 \text{ if and only if } \vec{u} = 0. \quad (2.20)$$

The important point about the decomposition (2.17) is that the velocity field is being separated into rotational and irrotational parts. The fact that  $\vec{u}$  is incompressible does not imply that part of the potential field cannot also be incompressible.

Substitute (2.17) into the continuity and momentum equations. We get

$$\frac{Dh}{Dt} + a^2 \nabla^2 \phi = \frac{\gamma}{\rho} (\phi - \dot{Q}) \quad (2.21)$$

and

$$\begin{aligned} & \text{grad} \left( h + \frac{\partial \phi}{\partial t} + \frac{1}{2} |\text{grad } \phi|^2 \right) \\ &= \vec{v} \times \vec{\omega} - \text{grad}(\vec{u} \cdot \text{grad } \phi + u^2/2) - \frac{\partial \vec{u}}{\partial t} \\ &+ T \text{ grad } s + \frac{\text{div } \vec{t}}{\rho} \end{aligned} \quad (2.22)$$

If we compare the last equation with the corresponding result in a homentropic-irrotational region (see (2.13)), we can interpret (2.22) as saying that the Bernoulli constant is no longer a constant. Thus, we define the fundamental unsteady enthalpy variable

$$\mathcal{H} = h + \frac{\partial \phi}{\partial t} + \frac{1}{2} |\text{grad } \phi|^2 \quad (2.23)$$

In regions  $H_1$  and  $H_2$ , this enthalpy is constant and equal to the appropriate value of the Bernoulli constant at infinity. In region  $R$ ,  $\mathcal{H}$  is a basic variable and is invaluable for the description of the source of sound. In the remainder of this work we shall refer to  $\mathcal{H}$  as the Bernoulli enthalpy.

With the definition (2.23), we can interpret (2.22) as an equation for the Bernoulli enthalpy. The integrability condition for (2.22) is that the curl of the right-hand side must vanish. This leads to the vorticity transport equation:

$$\begin{aligned} \frac{D\vec{\omega}}{Dt} &= \vec{\omega} \cdot \text{grad } \vec{u} + \vec{\omega} \cdot \text{grad}(\text{grad } \phi) - \vec{\omega} \nabla^2 \phi \\ &+ \text{grad } T \times \text{grad } s + \text{curl} \left( \frac{\text{div } \vec{t}}{\rho} \right) \end{aligned} \quad (2.24)$$

The Bernoulli enthalpy satisfies a Poisson equation obtained by taking the divergence of (2.22). After some manipulation of terms on the right-hand side, the resulting equation is

$$\begin{aligned} \nabla^2 \mathcal{H} &= - \frac{\partial^2 u^i u^j}{\partial x^i \partial x^j} - \frac{\partial^2}{\partial x^i \partial x^j} \left( u^j \frac{\partial \phi}{\partial x^i} \right) \\ &- \frac{\partial}{\partial x^j} \left( \frac{\partial u^j}{\partial x^i} \frac{\partial \phi}{\partial x^i} \right) + \text{div}(T \text{ grad } s) + \text{div} \left( \frac{\text{div } \vec{t}}{\rho} \right) \end{aligned} \quad (2.25)$$

We obtain the fundamental equation for the potential field in region  $R$  by substituting  $h$  from (2.23) into the continuity equation (2.21). We get

$$\frac{D}{Dt} \left( \frac{\partial \phi}{\partial t} + \frac{1}{2} |\text{grad } \phi|^2 \right) - a^2 \nabla^2 \phi = \frac{D\mathcal{H}}{Dt} - \frac{\gamma}{\rho} (\phi - \dot{Q}) \quad (2.26)$$

Finally, the pressure is obtained by substituting  $h$  into the state equation (2.6); i.e.,

$$p = p \left( - \frac{\partial \phi}{\partial t} - \frac{1}{2} |\text{grad } \phi|^2 + \mathcal{H}, s \right) \quad (2.27)$$

The basic theoretical development is complete. We summarize below the set of equations for  $\phi$ ,  $\mathcal{H}$ ,  $\vec{\omega}$ ,  $s$  and the pressure  $p$ :

#### The Sound Equations

$$\frac{D}{Dt} \left( \frac{\partial \phi}{\partial t} + \frac{1}{2} |\text{grad } \phi|^2 \right) - a^2 \nabla^2 \phi = \frac{D\mathcal{H}}{Dt} - \frac{\gamma}{\rho} (\phi - \dot{Q})$$

$$\begin{aligned} \nabla^2 \mathcal{H} &= - \frac{\partial^2 u^i u^j}{\partial x^i \partial x^j} - \frac{\partial^2}{\partial x^i \partial x^j} \left( u^j \frac{\partial \phi}{\partial x^i} \right) \\ &- \frac{\partial}{\partial x^j} \left( \frac{\partial u^j}{\partial x^i} \frac{\partial \phi}{\partial x^i} \right) + \text{div}(T \text{ grad } s) + \text{div} \left( \frac{\text{div } \vec{t}}{\rho} \right) \end{aligned}$$

$$\begin{aligned} \frac{D\vec{\omega}}{Dt} &= \vec{\omega} \cdot \text{grad } \vec{u} + \vec{\omega} \cdot \text{grad}(\text{grad } \phi) - \vec{\omega} \nabla^2 \phi \\ &+ \text{grad } T \times \text{grad } s + \text{curl} \left( \frac{\text{div } \vec{t}}{\rho} \right) \end{aligned}$$

$$\begin{aligned} \frac{Ds}{Dt} &= \frac{1}{\rho T} (\phi - \dot{Q}) \\ p &= p \left( - \frac{\partial \phi}{\partial t} - \frac{1}{2} |\text{grad } \phi|^2 + \mathcal{H}, s \right) \end{aligned} \quad (2.28)$$

### III. Discussion and Interpretation

The conceptual beauty of the sound equations (2.28) is perhaps not immediately evident. Here we discuss these equations in the light of certain self-evident truths and deductions based on previous models and studies of the aerodynamic sound problem.

First, we point out the obvious fact that the sound equations are exact. We have chosen a "natural" set of variables for the description of sound and have rewritten the fluid equations of motion in terms of those variables. No "model" of the sound problem is being proposed.

#### Bernoulli Enthalpy as the Source of Sound

There are two essential variables required for the description of sound in a rotational flow; namely, the velocity potential  $\phi$  and the Bernoulli enthalpy  $H$ . Only  $\phi$  is a true "acoustic" variable in the sense that it satisfies a wave equation capable of radiating sound energy to the far field. The "source" of the radiative sound field following a fluid element is the substantive rate of change of the Bernoulli enthalpy. Locally, the sound field (pressure) is a nonlinear function of the radiative  $\phi$  part and a second part due to the Bernoulli enthalpy. In the linear case, these two parts of the pressure field are additive via the state equation. The second part of the linearized pressure is directly proportional to the Bernoulli enthalpy. A direct analogy can be made between this "Bernoulli pressure" field and the concept of pseudosound (see below).

Our fundamental acoustic equation also shows that viscous dissipation and heat conduction (or bulk heat addition) are also direct sources (or sinks) of sound. For turbulent flows at moderate temperatures, the thermal source term should indeed be negligible. Lilley (5, Ref. 13 quoted therein) has argued that since turbulent dissipation is balanced by production and convective terms in a turbulent flow, it could be an important source of sound. At high wave numbers, on the scale of the viscous dissipation, this is most likely true. In fact, the entire right-hand side of our acoustic equation should vanish at high wave numbers and provide a high frequency cutoff of the radiative field. However, over a large part of the turbulent energy bearing spectrum the fluctuating Bernoulli enthalpy field should dominate as the primary source of sound. From another point of view, the dissipative source term in the acoustic equation is precisely the source of entropy. It is generally believed that except for very hot or very high Mach number turbulent flows that entropy is essentially constant along particle paths. Therefore, the assumption of local isentropic flow is consistent with the notion of neglecting the dissipative sound source in the acoustic equation. We remark

that these arguments should be valid even for Mach numbers well into the supersonic range.

An important feature of the sound equations is that the "Bernoulli source" of the radiative  $\phi$  field is compact; i.e., it is confined to the rotational turbulent flow regime.\* The Bernoulli enthalpy field must acquire an appropriate constant value on the dividing boundaries  $S_1$  between regions  $R$  and  $H_1$ . From a practical standpoint, it may be necessary to invoke this condition via asymptotic analysis. However, variations of the  $H$  field must not be permitted to permeate the homentropic region. In other words, the  $H$  field must reach a constant value on roughly the same scale as the vorticity or the entropy gradient at the boundary of a turbulent flow field. Crow (9) addressed this point in his critique of the Lighthill theory. It is unfortunate that Crow did not seem to recognize the compact property of the Bernoulli variable  $H$  that he introduced (see Section 5 of Ref. 9). A basic point of his work is that the Lighthill source term is not rigorously compact in the sense used here. Crow was able to show by asymptotic arguments, however, that one may consider the source to be essentially the local turbulent flow if the wave length is sufficiently large compared to the geometric extent of the turbulent region. Asymptotic arguments are not needed to argue the compactness of the source if the essential role of Bernoulli enthalpy as the source field is recognized.

#### The Source of Bernoulli Enthalpy

The Bernoulli enthalpy satisfies a Poisson equation (see (2.28)) subject to the boundary condition,  $H = \text{constant}$ , on the bounding surface of the rotational-turbulent flow. We interpret the right-hand side of the Poisson equation, presumed to be given, as the "source" of Bernoulli enthalpy and ultimately of the radiative sound field. For the sound production problem, the only equations needed in (2.28) are those for  $\phi$ ,  $H$  and the pressure  $p$ . The vorticity transport and entropy equations are, in general, important only for maintaining the basic turbulent flow. An implicit assumption is, of course, that the turbulent flow is more or less passive to the generated sound field. The intense sound problem would require a knowledge of the potential vortex and entropy interaction.

The source terms in the Bernoulli enthalpy equation are arranged in "approximate" order of increasing importance as we go from low speed to high speed flows. We use the term approximate because flow speed is not the only consideration in ordering the

\* The dissipative and heat sources are not rigorously compact. In fact, they are the terms that ultimately lead to dissipation of the far field.

source terms. For smooth low speed flows at ambient temperature, the dominant source term is the rotational-incompressible quadrupole:

$$\frac{\partial^2 u^i u^j}{\partial x^i \partial x^j} \quad (3.1)$$

By now we might call this the classic Lighthill quadrupole, it having first been pointed out by Lighthill nearly 25 years ago. A subtle difference between the source term (3.1) and the Lighthill source is the absence of density and hence the interpretation of the source as a stress. Here, the source is an energy density or pure kinematic quantity that produces the Bernoulli enthalpy field. The mechanism whereby the quadrupole Bernoulli source produces a radiating quadrupole sound field is also slightly more subtle than in the acoustic analogy of Lighthill. It is, in fact, more closely akin to the pseudosound concept of Ribner (see below). A key point is that the immediate near-field of a fluid element has the quadrupole character (3.1) independent of far-field arguments.

We must emphasize the fact that the  $\mathcal{H}$  field only has quadrupole or dipole character in a local sense. It is a true source with no far field. However, the substantive derivative of the local source field will produce a radiative field of the same order plus a higher order pole due to the convective derivative. This distinction in multipole terminology must be remembered when referring to the source and radiative fields.

The second and third terms on the right-hand side of the Bernoulli enthalpy equation are, respectively, quadrupole and dipole in character:

$$\frac{\partial}{\partial x^i \partial x^j} (u^j \frac{\partial \phi}{\partial x^i}) \quad \text{quadrupole} \quad (3.2)$$

$$\frac{\partial}{\partial x^j} (\frac{\partial u^j}{\partial x^i} \frac{\partial \phi}{\partial x^i}) \quad \text{dipole} \quad (3.3)$$

These source terms result from the interaction of the potential flow with the rotational flow. In at least two important cases, these terms could become more important than the basic incompressible quadrupole. First, if the flow becomes highly compressible, the potential flow field can become as significant as the rotational-incompressible field. Second, if the potential flow is highly curved as in the flow over abrupt geometric changes, there could be an intense dipole source via (3.3) as turbulence is convected through the curved region.

From a practical standpoint, the second case is probably much more significant than the first. In fact, it could well be the explanation of the intense sound field emitted from a blown flap, or a jet impinging on a solid surface. We can easily argue that the magnitude of the

dipole source term is at least comparable to (if not larger than) the two quadrupoles in the flow around sharp corners. The additional radiative efficiency of the dipole could then lead to the complete dominance of this turbulence/potential flow interaction term in the far field.

The last two terms in the Bernoulli enthalpy equation are also dipole sources:

$$\text{div} (T \text{ grad } s) \quad (3.4)$$

$$\text{div} (\frac{\text{div } \vec{\tau}}{\rho}) \quad (3.5)$$

The first term (3.4) is due to entropy gradients and/or fluctuations, while the second term (3.5) is due to fluctuating viscous stresses. Either high Mach number or some strong source of heating or cooling is necessary before the first term becomes of any consequence in the production of sound. We note, however, the clean separation of the entropy source from the basic kinematic sources in the present formulation. Fluctuating viscous shear stresses are known to be inefficient sound producers so that the last term, (3.5), is generally negligible. The flow around sharp corners may be an exception.

#### The Pseudosound Analogy - An Experimental Challenge

In his 1964 opus on the generation of sound by turbulent jets, Ribner (4) concisely summarizes his concept of a pseudosound near field and how it generates an acoustic far field. A direct analogy can be made with the present study.

To simplify the presentation, and following Ribner, we consider the linear isentropic version of the basic equations (2.28) for  $\phi$ ,  $\mathcal{H}$  and the pressure  $p$ ; i.e.,

$$\frac{\partial^2 \phi}{\partial t^2} - a^2 \nabla^2 \phi = \frac{\partial \mathcal{H}}{\partial t} \quad (3.6)$$

$$\nabla^2 \mathcal{H} = - \frac{\partial^2 u^i u^j}{\partial x^i \partial x^j} \quad (3.7)$$

$$p - p_0 = + \rho_0 \mathcal{H} - \rho_0 \frac{\partial \phi}{\partial t} \quad (3.8)$$

We identify  $\rho_0 \mathcal{H}$  as the pseudosound field,  $p^{(0)}$  and  $-\rho_0 \frac{\partial \phi}{\partial t}$  as the acoustic field,  $p^{(1)}$  of Ribner. Now operate on the acoustic equation (3.6) with  $-\rho_0 \frac{\partial}{\partial t}$  to get

$$\frac{\partial^2 p^{(1)}}{\partial t^2} - a^2 \nabla^2 p^{(1)} = - \frac{\partial^2 \rho_0 \mathcal{H}}{\partial t^2} = - \frac{\partial^2 p^{(0)}}{\partial t^2} \quad (3.9)$$

and from (3.7) we get

$$\nabla^2 p^{(0)} = - \frac{\partial^2 \rho_0 u^i u^j}{\partial x^i \partial x^j} \quad (3.10)$$



Equation (3.9) is Ribner's "Dilatation Equation" and (3.10) is his "pseudosound equation," thus completing the analogy.

The separation of the pressure into a near field and far field and postulation of a Poisson equation for the near field is a simple and perhaps useful intuitive concept. We feel, however, that the natural separation of the pressure field that results from the dichotomy of the velocity field is conceptually more attractive, and theoretically more sound. It is not clear that Ribner's postulated pseudosound  $p^{(0)}$  or the Bernoulli field  $H$  for that matter is one and the same as the near-field pressure given by the equation of state. The local acoustic pressure could well be the same order of magnitude as the Bernoulli pressure. Another objection to the pseudosound concept is that one cannot argue that the  $p^{(0)}$  field is rigorously compact like the  $H$  field.

The real challenge it seems is to devise an experiment whereby the two parts of the pressure field can be inferred. If either the rotational or the irrotational velocity field can be measured, then we can make a clean separation of the pressure with the present theory, at least if the rotational flow is homentropic. From the state equation (see the last of equation (2.28)), the pressure and the irrotational velocity field are sufficient to infer the Bernoulli enthalpy field. Also, it follows from (2.23) that a direct measurement of the perturbation enthalpy (or pressure in the homentropic case) and irrotational velocity field is sufficient to determine  $H$  and, hence, the source of sound. We ask the experimentalist: how do you measure the irrotational or rotational velocity field?

#### A Synopsis of the General Theory - Summary Remarks

The simplicity of the theory of sound proposed herein is a consequence of a few very basic concepts, some of which are well known. First, we have recognized (rather recalled) the stark simplicity of the description of sound propagation, including refractive effects, and the absence of aerodynamic sound sources in a homentropic-irrotational flow. It is characterized by a single scalar field, the velocity potential,  $\phi$ . Next, there is the central most important concept of Bernoulli enthalpy  $H$ , perhaps not so well known. The idea follows from a basic desire to extend the simple description of sound in homentropic-irrotational flow to a turbulent gas flow. By using the well-known Helmholtz decomposition of the velocity field, and the fundamental definition of Bernoulli enthalpy, the acoustic or radiative part of the sound field separates cleanly from the source field. The Bernoulli enthalpy becomes the sole source of radiative sound, while the more elementary kinematic and entropy sources are those that generate the Bernoulli field. Thus, the Bernoulli

enthalpy appears as a natural "link" between the complicated dynamics of a turbulent flow and the relatively simple potential acoustic field.

The primary aim of the present work has been to clarify some of the basic conceptual problems that have plagued the modern theories of sound in a real turbulent flow. Two of these problems have been resolved in our opinion. First, we have successfully isolated the radiative and source components of sound in the near field. Second, we have rigorously localized the "source" of aerodynamic sound, and without the aid of asymptotic analysis. Finally, we have shown that "source of sound" is synonymous with "source of Bernoulli enthalpy." The rotational and irrotational kinematics and the thermodynamic entropy fluctuations of a turbulent flow each produce a Bernoulli enthalpy field that in turn produces radiative sound. The process of vorticity transport plays a secondary role in sound production as long as the basic turbulent flow is passive to the overall sound field. Vorticity transport is, however, an important element of the sound interaction problem (see below, Section IV).

In our presentation of the "general theory of sound" we have stopped short of the mark - the computational problem of sound production lies ahead. We must carry out the necessary statistical analyses within the framework of our new theory and couple this to a "suitable" model of turbulent flow. The successful invariant turbulent model developed by Donaldson, et al. (17,18,19) over the past ten years should provide the basic ingredients to calculate the source of Bernoulli enthalpy and ultimately the radiative sound field. It is hoped that realistic calculations of sound production can be made in the near future.

#### IV. Interaction of Sound with a Two-Dimensional Shear Layer

In the previous section we focused our discussion and interpretation of the sound theory primarily on the problem of sound production by a turbulent flow. Here we consider the somewhat simpler problem of sound interaction with a mean shear flow. The computational utility of the Bernoulli field concept should become apparent.

For simplicity, we consider the problem of two-dimensional sound interacting with a parallel two-dimensional jet or wake. The important special cases of the free shear layer and boundary layer require a further generalization of the analysis given below. Suppose that the potential field and the incompressible rotational field (assumed to be homentropic) are perturbed by the impinging sound field. We let

$$\phi = \bar{\phi} + \phi' \quad (4.1)$$

$$\begin{aligned} \bar{u} &= \bar{u}_0 + \bar{u}' \\ \bar{u}'_0 &= \bar{u}'(y) \end{aligned} \quad (4.2)$$



$$\mathcal{H} = h_{\infty} + \frac{v_{\infty}^2}{2} + \mathcal{H}' \quad (4.3)$$

where  $\mathcal{H}'$  is the Bernoulli enthalpy defect and  $u(y)$  is the shear defect velocity; i.e.;

$$u(y) = U(y) - v_{\infty} \quad (4.4)$$

where  $U(y)$  is the total mean velocity profile, and  $v_{\infty}$  is the constant streaming velocity at  $\infty$ . Since the flow is two-dimensional, it is convenient to introduce a stream function for the rotational velocity field; i.e.,

$$u' = \frac{\partial \psi}{\partial y} \quad v' = -\frac{\partial \psi}{\partial x} \quad (4.5)$$

so that

$$\nabla^2 \psi = -\omega' \quad (4.6)$$

where  $\omega'$  is the single component of the perturbation vorticity. We emphasize that  $\psi$  and the Bernoulli enthalpy defect,  $\mathcal{H}'$ , must vanish when  $\omega'$  vanishes (see (2.20)). With the foregoing definitions it is a straightforward task to derive the following interaction equations:

$$\frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial t} \right) - a_{\infty}^2 \nabla^2 \phi = \frac{\partial \mathcal{H}}{\partial t} \quad (4.7)$$

$$\nabla^2 \mathcal{H} = -2U' \frac{\partial^2 \phi}{\partial x \partial y} + 2U'' \frac{\partial^2 \psi}{\partial x^2} - u(y) \frac{\partial}{\partial x} \nabla^2 \phi \quad (4.8)$$

$$\frac{\partial}{\partial t} \nabla^2 \psi - U'' \frac{\partial \psi}{\partial x} = -U' \frac{\partial \phi}{\partial y} - U' \nabla^2 \phi \quad (4.9)$$

where we have dropped the prime on perturbation quantities. The pressure is given by the linear relation

$$p - p_{\infty} = -\rho_{\infty} \frac{\partial \phi}{\partial t} + \rho_{\infty} \mathcal{H} \quad (4.10)$$

and the convective derivatives are, respectively,

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + U(y) \frac{\partial}{\partial x} \quad (4.11)$$

and

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + v_{\infty} \frac{\partial}{\partial x} \quad (4.12)$$

In the absence of a shear flow, the incident sound field (assumed to be given) is governed by the homogenous form of (4.7) and the pressure is given by the first term in (4.10). The Bernoulli enthalpy defect and the stream function are identically zero. If the shear flow is suddenly "turned on" we interpret the right-hand side of (4.8) and (4.9) as interactive source terms for  $\psi$  and  $\mathcal{H}$ . The last equation shows how the sound field generates a perturbation  $\psi$  field that, in turn, generates a Bernoulli  $\mathcal{H}$  field via (4.8). In addition, the sound field generates Bernoulli enthalpy directly via the first and last terms in (4.8). For strong shear

flows, we note that the last term in each of equations (4.8) and (4.9) is an order of magnitude smaller (with respect to the shear layer thickness) than the leading terms. These terms represent the direct interaction of the compressibility of the sound field with the mean flow.

The simplicity of the formulation of sound-shear interaction presented here is evident. It is a viable alternative to the Landahl-Lilley formulation (5), or the more recent momentum potential theory of Doak (10). We point out that interpretation of the interaction equations we have derived is somewhat different from that of the Landahl-Lilley equation (5,10). In the latter equation, the "acoustic" near field is all assigned to the pressure and a single third-order equation is derived. This equation exhibits no linear coupling between the pressure and any other perturbation quantity for a two-dimensional parallel shear layer. Thus, the conclusion is made that the "acoustic" mode does not couple to any other mode (vorticity, e.g.). The key point and our objection to the conclusion is the choice of pressure to define the acoustic mode. The same definition of acoustic mode was used by Chu and Kovaszny (13), although they did point out in a footnote that a further division could be made into radiative and nonradiative parts. We submit that this separation is absolutely essential if you want to calculate the source of sound that can be radiated to the far field. Thus, we define  $\phi$  to be the true "acoustic mode" and thereby obtain a linear coupling (via the mean shear and curvature) with the vortical mode. The Bernoulli enthalpy does not introduce any new mode, but greatly simplifies the interpretation of the sound vortex interaction process.

As a numerical example, we choose the simple problem of a plane sound wave impinging on a two-dimensional jet (see Fig. 2). For the purpose of illustration, we only consider the Bernoulli source and scattered sound field generated by the first term on the right-hand side of (4.8); i.e., the interaction of the vertical sound velocity component with the mean shear. Let

$$\phi = e^{-i(kx - \omega t)} (\phi_0 e^{-i\ell y} + \phi_s) \quad (4.13)$$

and

$$\mathcal{H} = \mathcal{H}_0(y) e^{-i(kx - \omega t)} \quad (4.14)$$

where  $\phi_s$  is the scattered sound field, and  $\phi_0 e^{-i\ell y}$  is the incident field. We calculate  $\mathcal{H}$  induced by the incident field. Substitute the incident part of (4.13) together with (4.14) into (4.8) with the last two terms omitted. We get

$$\frac{d^2 \mathcal{H}_0}{dy^2} - k^2 \mathcal{H}_0 = 2k\ell \phi_0 U'(y) e^{-i\ell y} \quad (4.15)$$

with the boundary condition that  $\mathcal{H}_0$  must

vanish at the two edges of the jet. We relax this condition slightly because we later consider a Gaussian velocity profile. Therefore, we solve (4.15) with the Green's function that decays exponentially on the scale of the incident wave number  $k$  outside the jet. After a single integration by parts, the final result for the Bernoulli enthalpy profile becomes,

$$\begin{aligned} \mathcal{H}_0(y) &= \phi_0 \ell (k - i\ell) e^{-ky} \int_y^\infty e^{-(k-i\ell)\eta} U(\eta) d\eta \\ &\quad - \phi_0 \ell (k + i\ell) e^{ky} \int_y^\infty e^{-(k+i\ell)\eta} U(\eta) d\eta \end{aligned} \quad (4.16)$$

Now consider the Gaussian profile

$$U(y) = u_0 e^{-(y/\delta)^2} \quad (4.17)$$

where  $u_0$  is the centerline jet velocity. The half-profile and its first derivative are plotted in Fig. 3. Also, we define the normalized variables

$$\kappa = k\delta, \quad \lambda = \ell\delta$$

$$z = y/\delta \quad (4.18)$$

Then, the normalized Bernoulli enthalpy profile becomes

$$\begin{aligned} \mathcal{H}^*(z) &= \frac{2}{\sqrt{\pi}} \frac{\mathcal{H}_0(\delta z)}{u_0 \cdot \ell \cdot \phi_0} \\ &= (\kappa - i\lambda) e^{\frac{(\kappa - i\lambda)^2}{4}} e^{-\kappa z} \operatorname{erfc}\left(-z + \frac{\kappa - i\lambda}{2}\right) \\ &\quad - (\kappa + i\lambda) e^{\frac{(\kappa + i\lambda)^2}{4}} e^{\kappa z} \operatorname{erfc}\left(z + \frac{\kappa + i\lambda}{2}\right) \end{aligned} \quad (4.19)$$

where

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt \quad (4.20)$$

is the complementary error function. Note that the source amplitude is linearly proportional to  $u_0$  and  $\delta$ .

If we ignore the (admittedly important) refractive effects in the wave equation (4.7), we can write down at once the formula for the transmitted and reflected waves. We have

$$\phi_t \sim -\frac{\delta e^{i\kappa z}}{2\ell a_\infty^2} \int_{-\infty}^\infty e^{-i\lambda z} (\omega - \kappa U) \mathcal{H}_0(\delta z) dz \quad (4.21)$$

and

$$\phi_r \sim -\frac{\delta e^{-i\kappa z}}{2\ell a_\infty^2} \int_{-\infty}^\infty e^{i\lambda z} (\omega - \kappa U) \mathcal{H}_0(\delta z) dz \quad (4.22)$$

In Fig. 4, the normalized Bernoulli enthalpy profile (4.19) is plotted for a total normalized wave number  $(\kappa^2 + \lambda^2)^{1/2} = 1$  at 30 and 60 angles of incidence  $\theta$ . The wave length is the half-width of the jet. The real part of the  $\mathcal{H}_0$  profile is a dipole, while the imaginary part is basically a monopole with a relatively flat distribution over the central part of the jet profile. The dipole is the dominant part of the source at low angles of incidence, while the monopole becomes dominant at larger incidence angles. We remark, however, that there is no interactive source of the type we have used as an example for either zero or 90-degree incidence.

## V. Concluding Remarks

A new and completely general formulation of the theory of sound in a turbulent gas has been presented. We have introduced the essential new concept of Bernoulli enthalpy. It appears to be the missing link between the complicated motion of a turbulent flow and the radiative acoustic field. The principal conclusions of our study are summarized below.

1. Sound in a turbulent gas can be characterized by two scalar fields: (1) the velocity potential  $\phi$ , and (2) the Bernoulli enthalpy  $\mathcal{H}$ . The acoustic or radiative near and far field is completely characterized by  $\phi$ . The Bernoulli enthalpy is variable only in the rotational turbulent region and is the sole source of the radiative field. The near-field sound pressure is, in general, a nonlinear function of the acoustic and Bernoulli fields. The far-field pressure depends only on the radiative  $\phi$  field.

2. The acoustic part of the sound field (near and far) depends only on the ir-rotational component of the velocity field.

3. The principal source of Bernoulli enthalpy in a low-speed turbulent flow that is not highly curved is the classic Lighthill quadrupole. In addition, when the mean potential flow is highly curved or compressible, there are rotational-irrotational interaction sources. In particular, there is a dipole term that is believed to be the dominant source of sound in the turbulent flow around corners and sharp edges. Also, the entropy source that is important in hot turbulent jets is a dipole that separates linearly from the kinematic sources in the present formulation.

4. There is a close analogy between the two-field representation of sound in the present work with the "pseudosound" formulation of Ribner (4). Our Bernoulli  $\mathcal{H}$  field is akin to Ribner's pseudosound near field. The separation of the two fields and derivation of their governing equations is much more direct with the present approach.

5. The familiar problem of two-dimensional sound interacting with a parallel shear flow is formulated in terms of the  $\phi$  and  $\psi$  fields and a stream function to characterize the perturbation vorticity. Radiative sound interacting with the mean flow shear and curvature produces perturbation vorticity and a resultant Bernoulli field. The vertical component of the sound field also interacts with the mean shear directly to produce a Bernoulli field. Also, the compressibility of the sound field is amplified by the shear defect velocity but, in general, the source will be weaker than either of those due to mean shear.

6. Numerical results are presented for the case of a plane sound wave impinging on a two-dimensional Gaussian jet. The Bernoulli enthalpy profile has the appearance of a dipole at low angles of incidence and a monopole at large angles.

7. The theory we have developed is not restricted to the study of acoustics. The  $\phi$  field describes the steady as well as the nonsteady potential flow. Thus, the general theory can be used to study a variety of potential vorticity interaction problems. It is hoped that specific results in this direction can be presented in the near future.

#### Epilogue

We have often heard the remark that "one cannot understand fluid mechanics without a working knowledge of the concept of vorticity." In conclusion, we offer the paraphrase, "One cannot understand the theory of sound in a rotational medium without a working knowledge of the concept of Bernoulli enthalpy."

#### Acknowledgements

This work was sponsored in part by the Air Force Office of Scientific Research under Contract No. F44620-75-C-0010.

The authors wish to thank their colleagues at A.R.A.P. for many helpful discussions and criticisms during the course of this work. In particular, the encouragement by and interaction with our colleague, Dr. Coleman duP. Donaldson, is most gratefully acknowledged.

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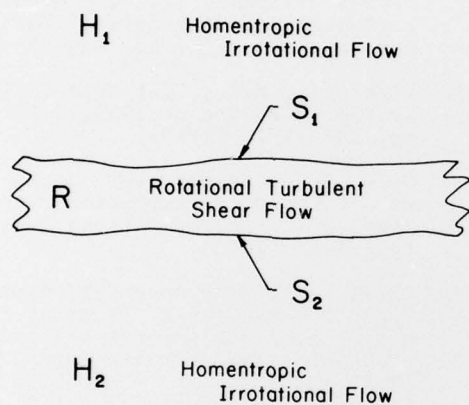


Fig. 1 Decomposition of the flow field

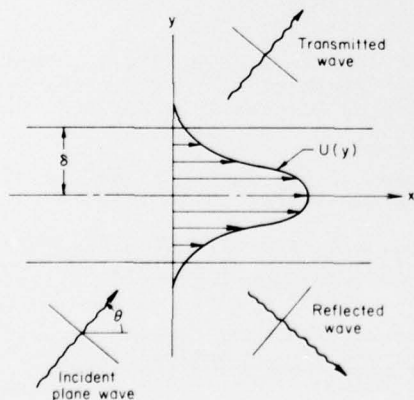


Fig. 2 Plane sound wave scattering from a plane jet

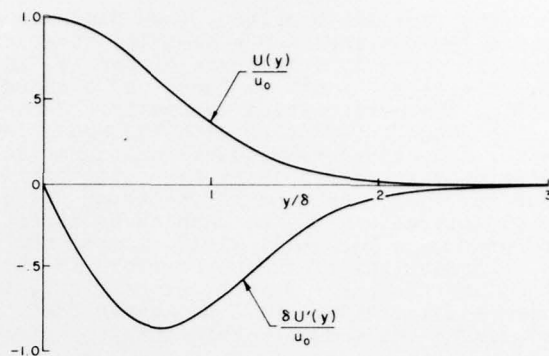


Fig. 3 Gaussian mean velocity and shear profiles

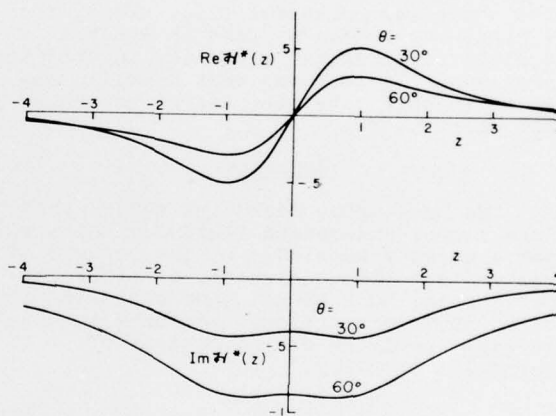


Fig. 4 Normalized Bernoulli enthalpy profile (plane sound on a plane jet)